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NOMOGRAM FOR THE DETERMINATION OF THE LIFETIME OF AN ARTIFICIAL EARTH'S SATELLITE

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SUMMARY

A graphic method is substantiated for an approximate evaluation of the lifetime of an artificial Earth's satellite taking into account the density variation of upper atmosphere layers in the course of the solar activity cycle. The nomogram proposed for the determination of an AES lifetime by this method is based upon known data on atmosphere density.

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Approximate formulas and tables are presented in ref.[1], which permit to compute rapidly and without cumbersome operations the lifetime of an AES, conditioned by their deceleration in the Earth's atmosphere. In all these calculations the atmosphere density is assumed to be constant during the entire lifetime of the AES for every altitude in correspondence with the CIRA-1961 model atmosphere.

Graphs, quite practical for their utilization in the determination of the lifetime of an AES are brought out in ref. [2]; they were obtained on the basis of numerical calculations with the aid of a computer. In this work the density of the atmosphere is also assumed to be constant in the course of the entire lifetime of the AES and corresponds to the ARDC-1959 model atmosphere.

The tables presented in ref. [3], which allow the finding of the lifetime of an AES for various laws of altitude variation of uniform atmosphere (H), are also not suited for the accounting of density variation in time, conditioned by solar activity variation.

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* NOMOGRAMMA DLYA OPREDELENIYA VREMENI SUSHCHESTVOVANIYA ISZ

When computing the AES lifetime, the density of the atmosphere is usually represented in the form

$$\rho(h) = \rho_{п} \exp\left(-\frac{h - h_{п}}{H}\right), \tag{1}$$

where h is the height of the satellite above ground, h_{Π} is height of the perigee of satellite orbit, ρ_{Π} is the atmosphere density in orbit's perigee, H is the height of uniform atmosphere in perigee, whereupon

$$H = f(h_{п}) = -[d \ln \rho(h_{п})/dh]^{-1}.$$

The AES lifetime $t_{сущ}$ for an atmosphere density given in the form (1), may be represented by the formula [1]

$$t_{сущ} = \frac{F[h_{п0}, e_0, H(h_{п0})]}{\sigma \rho_{п0}}, \tag{2}$$

where $\sigma = C_x S/G$ is the satellite's ballistic coefficient, C_x is the air drag coefficient, S is the area of satellite's midship cross section, G is the weight of the satellite, e_0, $h_{п0}$ and $\rho_{п0}$ are the initial values of orbit's eccentricity e, of h_{Π} and ρ_{Π}.

Function F in (2) may be represented for circular orbits in the form

$$F[h_{п0}, e_0, H(h_{п0})]_{e_0=0} = H(h_{п0}) \Phi_1(h_{п0}), \tag{3}$$

and for elliptical orbits at $h_{п0} = 200 \div 700$ *км* and $e_0 \gtrsim 0{,}05 \div 0{,}07$ (see [1]) in the form

$$F[h_{п0}, e_0, H(h_{п0})] = \frac{\Phi_2(h_{п0}, e_0)}{\sqrt{H(h_{п0})}}. \tag{4}$$

The distributions of atmosphere density in height in various years in the course of the solar activity cycle may be considered as different density models. For each i-th model, formulas, analogous to (1) - (4), will have the form

$$\rho_i(h) = \rho_{iп} \exp\left(-\frac{h - h_{п}}{H_i}\right), \tag{5}$$

$$t_{i\,сущ} = \frac{F[h_{п0}, e_0, H_i(h_{п0})]}{\sigma \rho_{i\,п0}}, \tag{6}$$

$$F[h_{п0}, e_0, H_i(h_{п0})]_{e_0=0} = H_i(h_{п0}) \Phi_1(h_{п0}), \tag{7}$$

$$F[h_{п0}, e_0, H_i(h_{п0})] = \frac{\Phi_2(h_{п0}), e_0)}{\sqrt{H_i(h_{п0})}} \tag{8}$$

Let us now admit that the satellite's lifetime $t_{\text{сущ}}$ and the variation in time of its height and orbit eccentricity are known for the law of density variation given by formula (1). Then, taking into account (2) - (4) and (6)- (8) we shall have for the density given by formula (5):

$$t_{i\text{сущ}} = t_{\text{сущ}} k_i q_i, \tag{9}$$

where

$$k_i = \frac{\rho_{\pi 0}}{\rho_{i\pi 0}}, \tag{10}$$

$$q_i = \frac{H_i(h_{\pi 0})}{H(h_{\pi 0})} \quad \text{at} \quad e_0 = 0, \tag{11}$$

$$q_i = \sqrt{\frac{H(h_{\pi 0})}{H_i(h_{\pi 0})}} \quad \text{at} \quad e_0 \gtrsim 0{,}05 \div 0.07. \tag{12}$$

For orbit eccentricities $0 < e_0 < 0.05 \div 0.07$ (for $h_{\Pi 0} = 200 \div 700$ km) one may not succeed in obtaining simple formulas, similar to (3), (4), (11) and (12). In this case, for approximate calculations it is possible to utilize in (9) the value $q_i = 1$. Indeed, as follows from (11) and (12), when passing from $e_0 = 0$ to $e_0 \approx 0.05 \div 0.07$, the dependence of q_i on H and H_i becomes inverse (H and H_i change places). This means that inside the interval $0 < e_0 < 0.05 \div 0.07$, q_i passes through the unity. Evidently, for values e_0 close to limits of the indicated interval, it would be more correct to utilize formulas (11) or (12), and not to assume $q_i = 1$.

A formula of form (9) may be also applied for the determination of the time Δt required for the satellite to descend from the altitude $h_{\Pi 0}$ to the given altitude h_Π in the case of density (5), if the corresponding time Δt for the case of density (1) is known. This means that the variation in time of satellite height for the model atmosphere (5) may be determined provided its variation for model (1) is known.

Assume that the curve in Fig.1 represents the dependence h_Π on the time $\underline{t}$ for the atmosphere density (1). It follows from (9) and from the above-said, that this very same curve will serve as"perigee trajectory" at density (5), if the scale along the time axis in Fig.1 is increased $k_i q_i$ times.

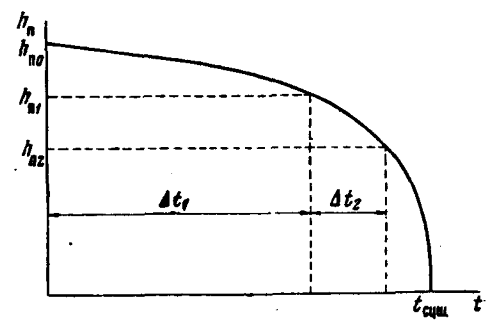

Fig.1

Assume that in the course of the time Δt_1 the density of the atmosphere corresponded to (5) at i = 1, and that during this time the orbit perigee height descended to the height h_Π. But in the course of the following time interval Δt_2 the density corresponded to (5) at i = 2 (the perigee descended to the height $h_{\Pi i}$) and so forth till i = j. The time of orbit perigee locating in the altitude range $h_{\pi 0} \div h_{\pi j}$ is then equal to

$$\Delta t(h_{\pi j}) = \sum_{i=1}^{j} \Delta t_i. \tag{13}$$

The values of Δt_i may be computed by formula (9) provided the time $\Delta t(i)$, during which the orbit perigee would be lowered from $h_{\pi(i-1)}$ to $h_{\pi i}$, and the density would not vary and would correspond to some other model, for example, (1) or $(i-1)$-th, is known.

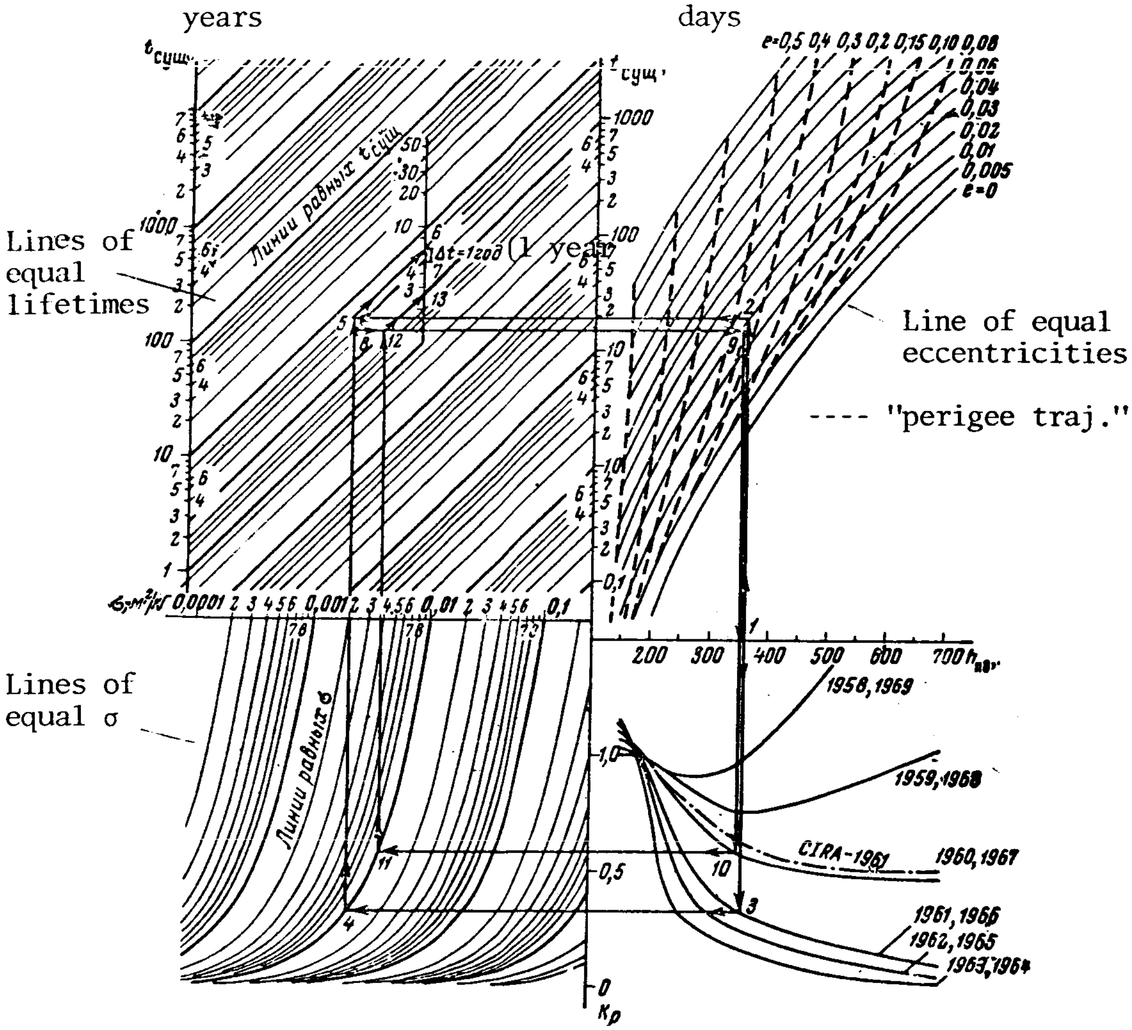

Fig.2. Example of nomogram use for the determination of AES's 1966 launching; $h_{\pi 0} = 360$ км $e_0 = 0{,}03$; $\sigma_0 = 0{,}01$

1. Operations 1 – 6 give $t_{\text{сум}} = 6.5$ years on the condition that ρ is constant during lifetime and corresponds to 1966.

2. Operations 7 – 13 serve for the determination of lifetime taking into account density variations during time intervals Δt and are repeated so long as h_π does not descend to the value of ~ 200 km. The values of Δt are chosen as a function of the required precision in the accounting of ρ variation by years. At the same time, $t_{\text{сум}} = \sum_i \Delta t_i$, where $\underline{i}$ is the number of cycles with operations 7 – 13. When $\Delta t_i = 1$ year for the example considered, $t_{\text{сум}} = 3.5$ years.

The described method of time determination, during which the satellite will descend to the assigned height, may be utilized to find its lifetime by postulating for the final perigee height $h_{\pi j} \approx 150$ km, inasmuch as at lower heights the motion of most of satellites already constitutes a fall to the Earth [1].

Figure 2 illustrates a nomogram allowing to rapidly determine the lifetime of an AES by a graphic method, taking into account the atmosphere density variations with time. The nomogram is constructed on the basis of numerical calculations by a method worked out in [2], using the model atmosphere ARDC-1959. The dashed curves in the upper part of the nomogram represent "perigee trajectories". They are intersected by solid lines of orbits' equal eccentricities. This part of the nomogram is constructed by analogy with the nomogram of [2]. In the lower right-hand part of the nomogram are figured the curves representing the values of the coefficient

$$k_\rho(h_\pi) = \frac{1}{k_i q_i}$$

for different years in the course of the solar activity cycle, whereupon it is postulated $q_i = 1$, i. e., the curves $k_\rho(h_\pi)$ in Fig.2 are related to the case $0 < e_0 < 0.05 \div 0.07$. For the cases $e_0 \approx 0$ and $e_0 \gtrsim 0.05 \div 0.07$, the curves $k_\rho(h_\pi)$ are shown in Figs. 3 and 4.

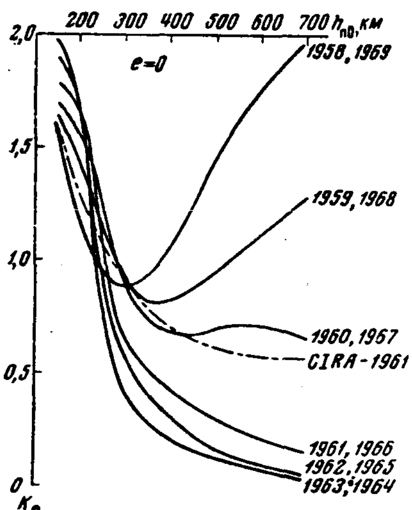

Fig.3

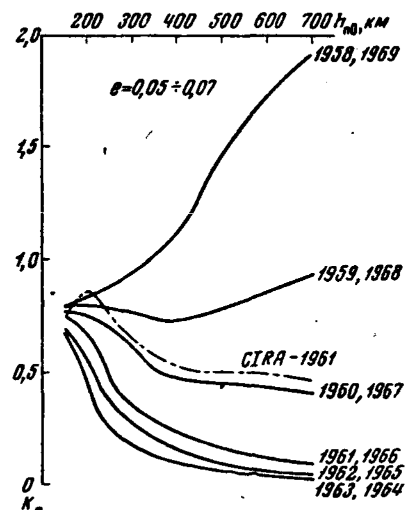

Fig. 4

The model atmospheres for the construction of the curves $k_\rho(h_\pi)$ were based on the data borrowed from ref. [4, 5]. At the same time, for 1963-1964 the curves were obtained by extrapolation because of the absence for these years at time of nomogram construction of complete data. It is also assumed that the atmosphere density will be the same in 1965, 1966 and 1962 as in 1961, and so forth. In the future the new data on the density of upper atmosphere layers will allow us to further refine the curves $k_\rho(h_\pi)$.

Plotted in the left-hand part of the diagram are the curves of equal ballistic coefficients of the satellite and the curves of equal lifetimes. The sequence of actions during nomogram utilization is indicated by arrows and numerals. The example brought out in Fig.2 shows that the variation of atmosphere density with time influences substantially the lifetime of satellites, and it must be taken into account.

In conclusion I wish to thank E. B. Kislaya for the help in shaping up the diagram.

**** T H E E N D ****

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